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Assessing the Effectiveness of Different Visualizations for Judgments of Positional Uncertainty

Grant McKenzie[a*], Mary Hegarty[b], Trevor Barrett[b] and Michael Goodchild[a]

[a]*Department of Geography, University of California, Santa Barbara*;
[b]*Department of Psychological & Brain Sciences, University of California, Santa Barbara*

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Many techniques have been proposed for visualizing uncertainty in geospatial data. Previous empirical research on the effectiveness of visualizations of geospatial uncertainty has focused primarily on user intuitions rather than objective measures of performance when reasoning under uncertainty. Framed in the context of *Google's blue dot*, we examined the effectiveness of four alternative visualizations for representing positional uncertainty when reasoning about self-location data. Our task presents a mobile mapping scenario in which GPS satellite location readings produce location estimates with varying levels of uncertainty. Given a known location and two smartphone estimates of that known location, participants were asked to judge which smartphone produces the better location reading, taking uncertainty into account. We produced visualizations that vary by glyph type (uniform blue circle with border vs. Gaussian fade) and visibility of a centroid dot (visible vs. not visible) to produce the four visualization formats. Participants viewing the uniform blue circle are most likely to respond in accordance with the actual probability density of points sampled from bivariate normal distributions and additionally respond most rapidly. Participants reported a number of simple heuristics on which they based their judgments, and consistency with these heuristics was highly predictive of their judgments.

Keywords: Visualization, uncertainty, positioning, human judgment, heuristics

1. Introduction

In 2004, a small technology company, *Where 2 Technologies*, specializing in digital mapping was acquired by Google and shortly thereafter the *Google Maps* platform was established. One year later, a *Google Maps for Mobile* Java application and mobile browser edition were released (Google 2007) and the smartphone-enabled public was introduced

*Corresponding author. Email: grant.mckenzie@geog.ucsb.edu

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to the *Blue Dot*. While this self-location indicator was not unique to Google Maps,[1] it was the public's first exposure to Google's *Blue Circle of Uncertainty* on a large scale. Although representations of positional uncertainty existed in GPS navigation devices prior to Google mobile maps design (Garmin's StreetPilot series for example), the company arguably brought the concept into the mobile mapping main-stream. Given the existence of this *Blue Circle of Uncertainty*, the purpose of this study is to examine how people make judgments about the positional information and uncertainty shown by the blue dot and how alternative visualizations of this information affect judgments.

Reporting one's location with a mobile device requires that the device in question afford some method of self-location. Among popular mobile device manufactures, there are currently three methods for geolocating a device, global navigation satellite system (GNSS) of which the Global Positioning System (GPS) is one system, Wi-Fi positioning, and cellular tower positioning. One aspect that differs between these technologies is the level of uncertainty with which geolocation is reported. Standard GPS has been shown to range in uncertainty from a few meters to hundreds of meters (Wing *et al.* 2005, Janowski *et al.* 2014) while Wi-Fi positioning and cellular positioning offer accuracy in the range of tens to thousands of meters (Wang *et al.* 2014, Bareth 2012, Zandbergen 2009). In the case of GPS positioning, there are certain measures of uncertainty that can be derived from the configuration of satellites above the horizon. Through multilateration (Chiang *et al.* 2012, Kennedy 2002), an individual's 3-dimensional location can be determined when a minimum of four satellites are in view of the receiver. As the number of satellites present in the calculation increases, along with the geometry for Geometric Dilution of Precision (GDOP), the accuracy in GPS positioning improves. A change in the GDOP can have a significant impact on the positional uncertainty of the receiver, in some cases by hundreds of meters (Teng and Wang 2015). Wi-Fi and Cellular technologies may be influenced by frequency interference, surface reflectance as well as number and placement of access points (Zandbergen 2012). Due to this large range in uncertainty both within and between technologies it follows that some representation of this uncertainty be presented to the end user. In Google Maps, this representation is an explicit blue circle (Figure 1) superimposed on the map. The addition of the blue circle glyph to the existing map represents an extrinsic approach to representing geospatial uncertainty, which means that the blue circle is a *layer* on top of the map that is adjusted independently of the underlying base map. In contrast, an intrinsic approach would rather alter or distort existing features of the map (Howard and MacEachren 1996, Gershon 1998).

While this blue circle represents uncertainty in the location of the mobile device, the method by which the circle is generated is never fully explained. Google's documentation states that "...at times, you may see the dot surrounded by a light blue circle. This indicates that there is some uncertainty about your location."(Google 2014b) and "...you may be anywhere within it"(Google 2014a). This then begs several questions. What exactly does the blue circle of uncertainty represent? The use of an opaque fill with a *hard boundary* suggests that the circle depicts some positional confidence interval (most likely 95%). Google's intentions aside, what does an individual perceive the circle of uncertainty to represent? Does the average mobile map user interpret the data appropriately given how uncertainty is visually presented and does the interpretation depend on the visualization method?

In the next section we review the relevant literature on the topic of positional uncertainty and follow it with an introduction to our study and research contribution.

[1]GPS-enabled navigation devices were well established prior to the release of Google Maps for Mobile.

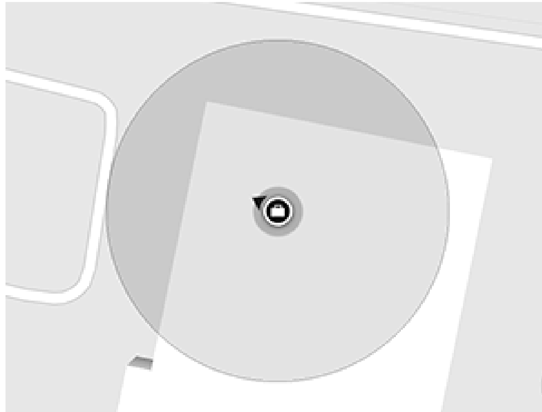


Figure 1.: Circle of uncertainty currently shown on Google Maps (normally shown in blue).

1.1. *Related Work*

Over the last thirty years, researchers in geographic information science have designed and evaluated visualization of uncertainty in several different specific contexts (MacEachren *et al.* 2005, Kinkeldey *et al.* 2014). For example, Wittenbrink *et al.* (1996) developed a vector glyph type to aid in the visualization of uncertainty in wind and ocean currents and Grigoryan & Rheingans (2004) proposed a technique for visualizing uncertainty in surfaces through a point-based approach to represent the surface, offsetting the points to represent uncertainty in the data. In the field of remote sensing researchers have recommended various methods for improving understanding and perception of uncertainty with respect to satellite imagery (Gahegan and Ehlers 2000, Bastin *et al.* 2002, Blenkinsop *et al.* 2000). For example, Drecki (2002) proposed and tested methods for visualizing uncertainty in land cover classification of satellite imagery, including a new method which altered the size of squares based on uncertainty of the data. The results of a participant study found that while color saturation and the squares technique were the preferred methods for portraying uncertainty, the opacity technique was the most effective. A study by Leitner & Buttenfield (2000) focused specifically on visualizing attribute uncertainty on thematic maps, exploring the effect of varying *Value*, *Texture* and *Saturation* in spatial attribute uncertainty on correctness, time and confidence. In the context of constructing choropleth maps, Schweizer and Goodchild (1992) proposed the use of color for data quality and uncertainty depiction. These studies demonstrate that particular visual depictions of uncertainty work in specific cases, but do not provide a consensus on which representations of visual uncertainty work best in general. In addition, while much of the geospatial research focuses on visualization for spatial data, research related to self-localization uncertainty is particularly sparse.

In other investigations, researchers in geographic information science and related disciplines have developed taxonomies of *visual* representations of uncertainty. In 1991, Buttenfield and Beard (1991) held a specialist meeting focusing on visualizing the *quality* of spatial information and outlining the need for further research in this area. Early research in the area of geospatial data quality (Buttenfield and Weibel 1988) proposed the development of a framework for categorizing components of *data quality* and made suggestions for their cartographic representation including adjusting the size, shape, color saturation, color value and texture based on the quality and type of geospatial data being presented. Subsequent work by MacEachren (1992) focused on addressing the differences between data quality and uncertainty as well as the role of cartographic symbols and types of user interfaces employed in visualizing uncertainty. This work builds on Bertin's (1983) work on *Semiology of Graphics*, specifically addressing how graphics can be used to represent geospatial data uncertainty. Thomson *et al.* (2005) provided an expanded typology with the goal of providing a consistent basis for constructing uncertainty visualizations. Framed with examples from the national security field, this typology presents classes for analyzing uncertainty and a framework for refining those classes based on specific tasks. Pang *et al.* (1997) proposed a number of general uncertainty visualization approaches ranging from adding *glyphs* to *animation* and *sonification*, and discussed the appropriateness of each method, based on the type of data presented. While visualizations of geospatial uncertainty have been classified and evaluated through many different scenarios, little research has focused on methods applicable to *positional* uncertainty of a user-centered mobile device.

More recently, researchers have begun to examine how people interpret and understand various visual representations of uncertainty in objective empirical tests. For example, work on the interpretation of confidence intervals and error bars on graphs found that scientists have "severe misconceptions" about how error bars relate to statistical significance (Belia *et al.* 2005). The authors of this work suggest less ambiguous graphical conventions and better guidelines for statistical researchers. Another recent study (Padilla *et al.* 2015) explored the use of various visualizations of uncertainty on perceived accuracy of weather forecasts. Participants were presented with different visualizations (glyphs) representing uncertainty of single scalar values along with supplemental statistical information, with the goal of determining which visualizations were more influential. The authors found that there were differences in how decisions were made between spatial (e.g., error bars) versus non-spatial glyphs (e.g., using color hue to represent uncertainty), but found no difference among the spatial glyphs themselves. Our work expands on this study, altering the task and expanding the dimensionality to include positional uncertainty. MacEachren *et al.* (2012) examined the *intuitiveness* of a number of sign visualizations for communicating types of uncertainty. While this work revealed that fuzziness and graded point size were the most intuitive, it did not assess how the visualizations affected task performance, such as decision making. Therefore it did not account for the possibility that the most intuitive symbols are not necessarily the best for supporting accurate task performance. Map-users often desire realistic and complex maps, but in reality, simpler maps are often the most effective at conveying the requisite information (Smallman and John 2005, Hegarty *et al.* 2012), a phenomenon known as *Naïve realism.* This highlights the need for objectively testing the effectiveness of uncertainty visualizations for supporting accurate judgments, and not just relying on users' intuitions of effectiveness. Here we explore these questions in the context of the *blue circle of uncertainty*, filling a gap in research related to judgments of positional uncertainty.

1.2. *The Present Study*

In this paper, we present a novel experiment to better understand the effectiveness of different visualizations in depicting positional uncertainty. Our task is presented in the context of a mobile application where GPS satellite location readings produced different levels of uncertainty. Given a known location and two smartphone estimates of that known location, participants were asked to judge which smartphone produced the better location reading, taking uncertainty into account. It should be noted that this is a hypothetical mobile application and participants were not tested in the actual environment, but rather in a laboratory setting. The study focuses on determining which visual aspects are most beneficial to individuals when asked to make a judgment based on uncertain data.

With this in mind, four visualizations (glyphs) for representing geospatial uncertainty are presented. In all cases the field of uncertainty with regard to each smartphone is modeled as a bivariate normal distribution, with equal variances and zero covariance. As a baseline, the standard two-dimensional blue circle of uncertainty with uniform opacity and border at the 95% confidence interval is presented both with and without a centroid present (in Google's representation, the centroid is present, see Figure 1). In addition, based on the suggestion that faded glyphs may be better suited for visualizing uncertainty (MacEachren *et al.* 2012), a two-dimensional opacity fade glyph is shown in comparison, again both with and without the centroid. The motivation for presenting the visualizations both with and without the centroid is that Google's position visualization on mobile devices currently presents a centroid marker. To the best of our knowledge, no research has examined effects of the presence of this centroid on positional uncertainty judgments.

To be clear, the term *uncertainty* is employed throughout this paper to refer to "the lack of certainty" differentiating it from the term *accuracy* which we use in reference to the degree to which a participant correctly performs a task. Furthermore, precision (the repeatability of a measure) is rarely mentioned in this work and only used when participants themselves mention the term or it is used in the relevant literature.

When tasked with a judgment about uncertainty, and without explicitly stating the probabilistic distribution of the data, it has been found that people tend to employ *heuristics* (Tversky and Kahneman 1974). Heuristics are simple decision algorithms or "rules of thumb" that are based on only a subset of the available information. During decision making, people often rely on one or more simple heuristics rather than one general-purpose calculus of rationality (Bröder and Schiffer 2003). For example, in one heuristic, known as "Take the Best," people make decisions based on ordering their choices on the basis of only one attribute, and ignore other properties (Gigerenzer 2007). In addition to examining the effectiveness of different visualizations, we investigate what heuristics participants use in our judgment task and whether these heuristics change as the visual representations of uncertainty are altered.

With this research background, the purpose of the study is to address the following research questions which make up the scientific contribution of this work:

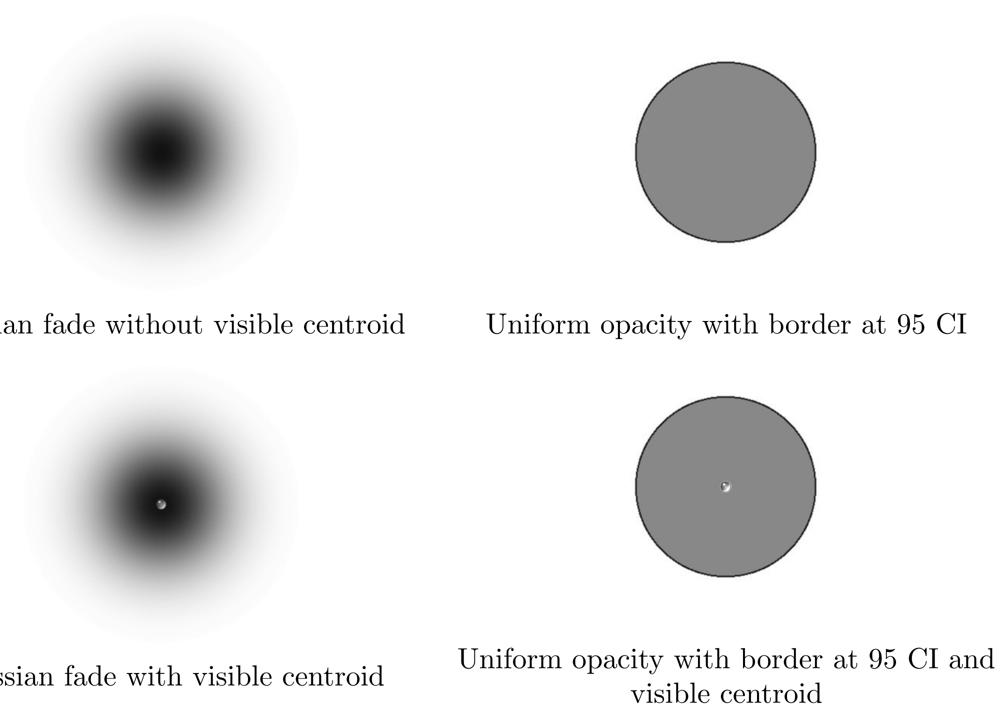
> $\mathcal{R}1$: Does the way in which uncertainty is visually depicted influence participants' judgments of positional uncertainty? Specifically:
>
> $\mathcal{R}2$: Is the use of different heuristics for uncertainty assessment dependent on the visual representation of uncertainty? And if so, which visualization formats lead to the adoption of which heuristics?
>
> $\mathcal{R}3$: Assuming a bivariate normal distribution of uncertainty, which visual aspects produce the most accurate assessments when asked to make a judgment based on uncertain data?
>
> $\mathcal{R}4$: Which visual aspects produce the fastest assessments when asked to make a judgment based on uncertain data?

We hypothesize that the heuristics that participants adopt will depend on the visual stimuli (glyphs) that are presented to them. For example, we predict that the inclusion of a centroid marker will make the center of the distribution more salient, biasing students to make their judgments based on *perceived distance* to the glyph's centroid. We also predict that the Gaussian fade glyphs will produce more accurate judgments because they provide more detailed information about the probability density function, and previous research (MacEachren *et al.* 2012) has indicated that faded glyphs may be better suited for visualizing uncertainty.

2. Methods

2.1. *Participants*

The participants were 110 undergraduate students (71 female) in an introductory psychology class at a research university, who participated in return for course credit and were randomly assigned to one of four visualization conditions. There were 28 participants in the *uniform circle* condition, 27 in the *uniform circle plus centroid* condition, 29 participants in the *Gaussian fade* condition and 26 participants in the *Gaussian fade plus centroid* condition (see Figure 2). Four additional participants were omitted due to extremely long response times or failure to follow directions.



Gaussian fade without visible centroid

Uniform opacity with border at 95 CI

Gaussian fade with visible centroid

Uniform opacity with border at 95 CI and visible centroid

Figure 2.: Four representations of positional uncertainty.

2.2. *Materials*

Stimuli were displayed as 1600 × 800 pixel images on a 120hz Samsung SyncMaster 2233 LCD monitor using SuperLab 4.5 experiment software. On each trial, participants were shown two maps of the same area with the same *known location* marked with an ×, (see sample trial in Figure 3) and estimates of the *known location* from two different smart phone displays. Their tasks were to decide which smartphone produced the best location reading for the known location.

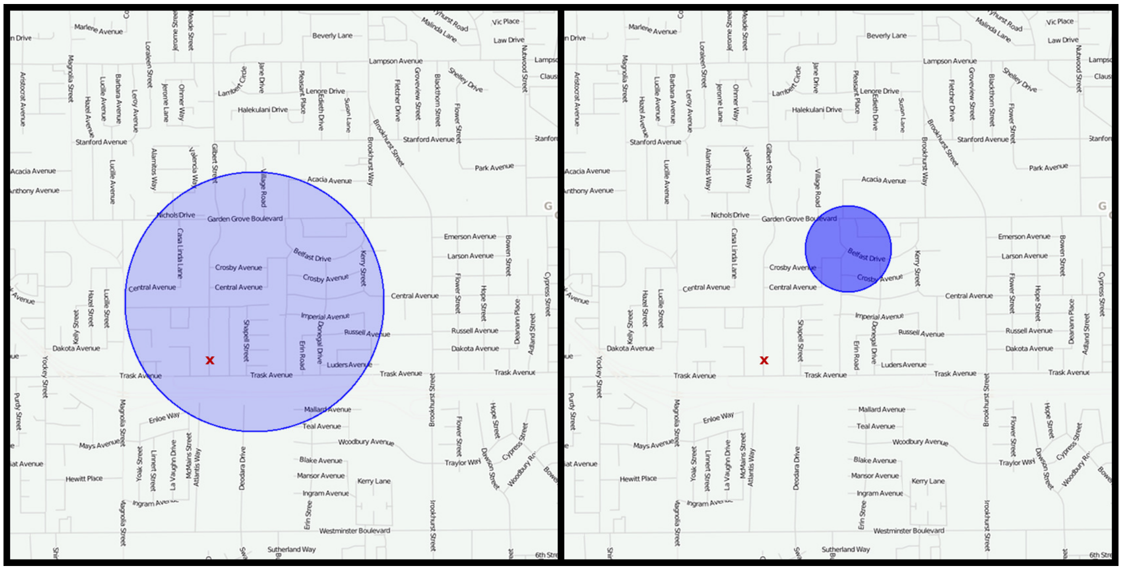


Figure 3.: Example trial showing two maps of the same area each with the same known location ("×") but with different probability distributions.

To create the stimuli, eight unique bivariate normal probability distributions were constructed to represent different amounts of positional uncertainty and divided into four distribution pairs (Figure 4). Each pair made up the two mobile device location readings for a trial. To ensure consistency with respect to existing *Google* maps displays, the bivariate normal probability distributions were circular (*var(length)=var(width)* and *cov(length,width)=0*). In each pair of distributions, the mean centers of the two distributions were spaced at 100 pixels apart in image space, equivalent to 500 meters on the ground (Euclidean distance). The angle between the two distributions for two of the pairs was 45 degrees while the other two were aligned at -45 degrees (as shown in Figure 4). The standard deviations on which the eight distributions glyphs were created ranged from 95m to 890m, with a total map area of 16 square km.

For each pair of distributions, eight unique known locations were drawn from the probability distributions (Figure 4). The known locations were chosen such that they had approximately equal likelihood of being sampled from the larger and the smaller distribution across trials. While all eight known locations are shown in the figure, participants were only shown a single known location on each trial and the distributions were shown separately, placed side by side (see Figure 3).

Four visual representations of bivariate normal distributions (with equal variances and zero covariance) were constructed as shown in Figure 2. Two of these representations express uncertainty through a Gaussian fade from opaque to transparent while the other two depict spatial uncertainty with uniform opacity, constrained at the 95% confidence interval with a solid border. The *fade* glyphs were created through the use of the *libtiff* library by generating a constant color image (HEX #0000FF) and introducing a Gaussian faded alpha channel starting from the center of the image and fading over a specified radius. The uniform opacity representations were generated without the Gaussian fade function and follow the standard "you-are-here" uncertainty concept communicated

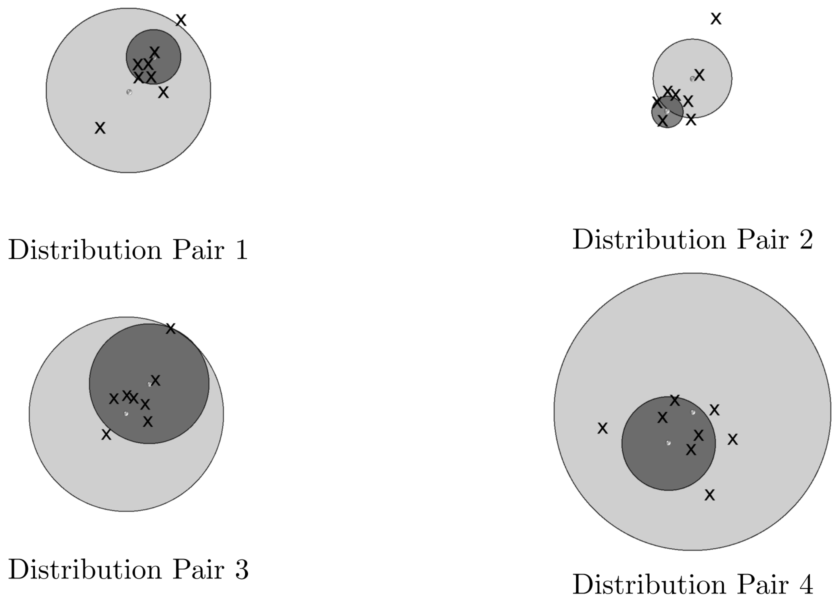


Figure 4.: Four distribution pairs constructed from eight bivariate normal distributions. Though shown here with opaque bordered circles and centroids, these same distribution pairs were also constructed with Gaussian faded glyphs both with and without visible centroids.

through the current *Google Maps for Mobile* application. Again, using the same color as the faded glyphs, the difference in opacity between glyphs depends on the radius, with a radius of 95m producing an opacity of 90% and a radius of 890m reducing the opacity to 30%. A Gaussian function with a small radius (standard deviation) decreases at a faster rate than a Gaussian function with a larger variance, and this was visualized by the decrease in opacity of the fade glyphs. In addition, two representations (one of each type) include a *blue dot* marking the center of the distribution and two do not.

The task was presented in the real-world context of judging the relative uncertainty of the location-based information provided by two new mobile devices. A total of 128 trials were produced for each of the four visualization formats by displaying the 32 trials (four distribution pairs × eight known locations) in four orientations each, (original, rotated, flipped and flipped & rotated). A minimalistic base map was selected from suburban Phoenix and constructed from simple road data collected via *OpenStreetMap*.

A post-study on-line questionnaire asked participants about their statistical background, whether they owned a mobile device, and if so, how often they used a navigation application. Additionally, in an open ended question, participants were asked to report any approaches or strategies they used in making their judgments.

2.3. *Procedure*

After giving informed consent, each participant was randomly assigned to one of the four visualization conditions, and read the task instructions, which are given in the Appendix. The participant was provided with an opportunity to ask questions before completing 128 trials at his or her own pace. On each trial, participants saw two maps of the same area (same center and scale) side by side (see Figure 3). The same known location was shown on the two maps and the visualization of one probability distribution was shown on each map. They indicated which distribution produced the more accurate location reading for the known location by pressing a key marked L for the left distribution or R for the right distribution. Finally, the participants completed the post-task questionnaire.

2.4. *Coding and Scoring*

2.4.1. Coding of Questionnaire Responses

Participants' responses to the post-task questionnaire were coded to reveal what stimulus properties and heuristics informed their judgments. Participants based their judgments on five primary properties, as follows. Two raters independently coded the responses stated by the participants and agreed in 93 of these cases (95% inter-rater reliability). Multiple strategies were coded for and discrepancies were resolved by consensus.

- **Distance**. Many participants based their judgments on the distance from the known point to the blue region showing the cell phone's predictions. Some participants explicitly stated a distance heuristic, that is, choosing the option in which the known point was closest to the glyph or center of the glyph, but most just mentioned distance as the basis of their predictions.
- **Size**. Several participants reported that they based their judgments on the size of the glyph (circle or Gaussian fade) with some explicitly stating that they chose the smaller glyph, indicating less uncertainty.
- **Darkness**. In some cases participants referred to the darkness (opacity) or color (more blue) of the glyph rather than its size; in the visualizations, opacity covaried with size so that smaller glyphs appeared darker.
- **Derived Variables** In some cases, rather than referring to the visual variable itself (e.g., size, opacity) participants referred to an interpretation of this variable, such as choosing the option in which the blue region was more *precise*, *accurate*, or *certain*. In these cases it was typically not possible to determine which visual variables they had used to make their judgments.
- **Containment**. A number of participants reported that they made their judgments based on whether the known point was inside or outside the blue region. When the known point was inside one blue region but outside the other, participants stated that they chose the circle (distribution) that contained the known point. This heuristic was applicable on 52 of 128 trials in which the known point was inside one circle and outside the other.

Several (55) of the participants mentioned more than one of the properties (e.g., "I looked at the distance between the blue dot and the red × and I also looked at whether the red × was inside the blue area and how big the blue area was"). Some stated a clear hierarchy among the heuristics, for example, "I tried to chose [sic] the one in which the blue dot was closest to me, unless the blue circle did not encompass my true location. In that case I would choose the one that placed me within the larger blue circle, even if I was further away." Finally, some participants (8) did not state a reason or it was not possible to code their answer (e.g., "I looked at the circles that surrounded the blue dot and ×").

2.4.2. *Coding of Responses to Experimental Trials*

Responses to the experimental trials were coded for consistency with three heuristics, based on distance (choose the distribution whose centroid is closest to the known location), size/darkness (choose the smaller/darker glyph) and containment (if the known point is inside one blue region but outside the other, choose the one it is inside). We also computed a measure of judgment accuracy based on *relative probability density*, as follows.

Given two uncertainty regions, U_1 and U_2, a bivariate normal distribution was generated for each U centered at location x_u, y_u with standard deviation σ_u. The probability density value can be extracted from the probability distribution function for any randomly selected point $p(x, y)$. Note that in this case the distribution means, x_u and y_u were centered at 0, 0 and the correlation equated to zero, resulting in the revised bivariate normal distribution equation shown in Equation 1.

$$p(x, y) = \frac{1}{2\pi\sigma_u^2} \exp\left(-\frac{1}{2}\left[\frac{x^2}{\sigma_u^2} + \frac{y^2}{\sigma_u^2}\right]\right) \tag{1}$$

Provided a known point, $p(x, y)$, probability density values were extracted from within each of the uncertainty distributions, U_1 and U_2. The *statistically* correct answer for each trial was defined as the distribution for which the probability density value of the known point was greater. The measure of *overall accuracy* then represents the proportion of trials on which participants chose this statistically correct distribution.

3. Results

3.1. *Demographics*

Most ($N = 104$, 94.5%) participants reported that they owned a smartphone and most ($N = 105$, 95.5%) stated that they had used a navigation application. Reported frequency of using a navigation application ranged from once a month or less (39 participants) to weekly (37 participants) to daily (34 participants). The majority of participants (69, 62.7%) reported having taken a statistics class. Neither frequency of using a navigation application, $F(2, 107) = .53$, $p = .59$, statistics training $t(107) = 1.38$, $p = 13$, nor gender $t(107) = .94$, $p = .35$ had significant effects on accuracy.

3.2. *Effects of Visualization on Heuristics Reported*

Chi-square tests of independence were conducted to evaluate the relationship between visualization conditions and the stimulus properties or heuristics that participants used to make their judgments. *Distance* was the most commonly mentioned property in all four visualization conditions (Table 1) and mention of this property did not significantly differ across conditions. Participants were more likely to mention *Size*, χ^2 (1, N=109) = 4.99, p = .03, and *Containment*, χ^2 (1, N=109) = 4.69, p = .03, as a basis for their judgment in the circle conditions than in the Gaussian fade conditions, χ^2 (1, N=113) = 5.02, p = .03. In contrast they were marginally more likely to mention color or darkness, χ^2 (1, N=109) = 2.83, p = .09, and more likely to mention derived variables (precision, accuracy, certainty, etc.), χ^2 (1, N=109) = 7.99, p = .005, in the Gaussian fade conditions than in the Circle conditions. Finally they were more likely to mention color or darkness, χ^2 (1, N=109) = 4.75, p = .03 when the centroid was not marked than when it was marked.

Heuristic	Uniform Circle	Circle + Centroid	Gaussian Fade	Fade + Centroid
Distance	21	17	22	15
Size	17	10	9	7
Darkness	3	1	8	2
Derived Variables **(precision, accuracy, etc.)**	0	2	7	5
Containment	12	9	3	8

Table 1.: Number of participants in each condition who mentioned each of the properties as the basis for their decision.

Next we examined whether participants judgments were based on the properties and heuristics that they mentioned in their strategy reports. To do this we used t tests to compare consistency of judgments with each heuristic (e.g., "choose the closest glyph") of participants who did and did not mention the relevant stimulus property (e.g., distance). Students who mentioned distance chose the closest glyph more often (on .71 of trials, SD = .11) than those who did not mention distance (.65, SD = .13, $t(107)$ = 2.33, p = .02). Similarly, students who reported responding on the basis of size, darkness or certainty, chose the smaller blue region more often (.63, SD = .19) than those who did not mention these properties (.55, SD = .12). Finally, students who reported using the containment heuristic were more likely to respond consistently with this heuristic (.83, SD = .15) than students who did not report using containment (.64, SD = .26; $t(95.6)$ = 4.60, $p < .001$, equal variances not assumed). In sum, participants' response choices were generally consistent with their reported heuristics.

3.3. *Effects of Visualizations on Consistency of Judgment with Heuristics*

Figure 5a shows the proportion of trials on which students responded according to the distance heuristic, that is, they chose the distribution for which the known point was closer to the centroid of the distribution. Responses were consistent with this heuristic for the majority of trials. A 2 (glyph: circle vs Gaussian fade) by 2 (centroid: absent vs. present) $ANOVA$ indicated that as predicted, students were more likely to respond according to the distance heuristic when the centroid was marked than when it was not marked F (1, 106) = 7.98, p = .006, η_p^2 = .07. The type of glyph (circle or Gaussian fade) did not affect consistency with the distance heuristic ($F < 1$), nor was there a significant interaction of glyph and centroid marking ($F < 1$).

Figure 5b shows the proportion of trials on which participants chose the distribution with the smaller/darker glyph (indicating less uncertainty). A 2 (glyph) by 2 (centroid presence) ANOVA indicated an interaction of visualization with centroid marking. Responding according to the size (darkness) heuristic was most prevalent for participants who viewed the Gaussian fade glyphs with the centroid marked, F (1, 106) = 4.27, p =

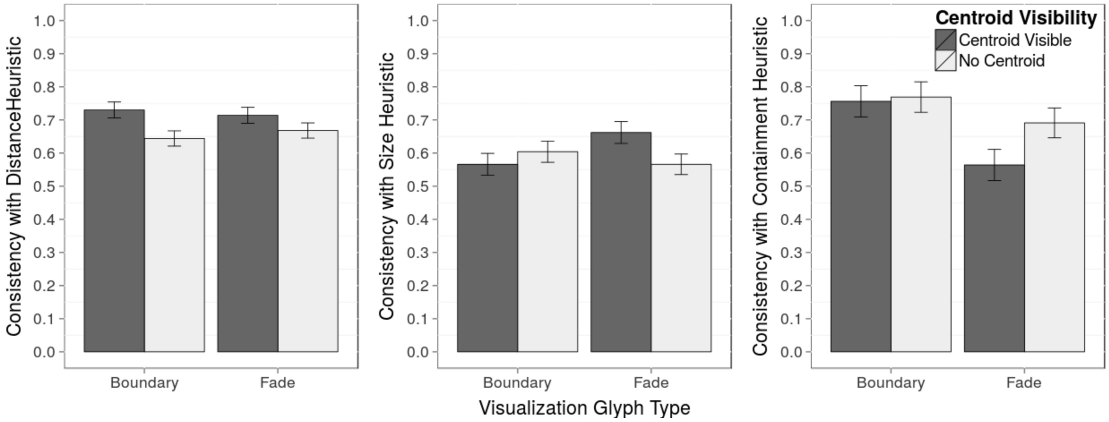


Figure 5.: Portion of trials in which participants responded according to (a) Distance, (b) Size, (c) Containment heuristics.

.04, $\eta_p^2 = .04$. Neither Visualization ($F < 1$) nor centroid marking ($F < 1$) had significant main effects on this variable.

Figure 5c shows the proportion of trials on which participants responded according to the containment heuristic when it applied, that is, the known point was inside the blue region for one of the distributions, but outside the blue region for the other. A 2 (glyph) by 2 (centroid presence) ANOVA indicated that students who viewed the circle glyphs were more likely to respond in terms of this heuristic than participants who viewed the Gaussian fade glyphs, F (1, 106) = 8.6, p = .004, $\eta_p^2 = .08$. Marking the centroid, F (1, 106) = 2.30, p = .13 and the interaction of glyph with centroid marking F (106) = 1.53, p = .22, did not have significant effects on this variable.

In summary, our analyses of reported heuristics and consistency of responses with heuristics address our first two research questions and indicate that participants judgments of positional uncertainty are influenced by how uncertainty is visually depicted. Specifically, when uncertainty is visualized as a uniform circle, participants are more likely to base their judgments on containment and when the centroid is marked, they are more likely to base their judgments on the distance heuristic.

3.4. *Effects on Performance*

3.4.1. Effects of Visualization on Accuracy of Judgments

Figure 6a shows the measure of accuracy, computed over all trials in the experiment, for participants who viewed the four different glyphs. A 2 (glyph: circle vs fade) by 2 (centroid present vs abstent) ANOVA on these data indicated that participants were less accurate when the uncertain region was represented by a Gaussian fade (Mean Proportion Correct =.64, SD = .13) than when it was shown by a uniform circle (.76, SD = .14), F (1, 106) = 23.28, $p < .001$, $\eta_p^2 = .18$. Displaying the centroid of the uncertain region also decreased accuracy from .74 (SD = .16) to .66 (SD = .13), F (1, 106) = 10.17, p = .002, $\eta_p^2 = .09$. The interaction of glyph type and centroid was not significant, $F <$ 1, indicating that the effects of the two factors were independent. These data address our third research question and indicate that, contrary to prediction, the uniform circle without centroid marking produced the most accurate responses.

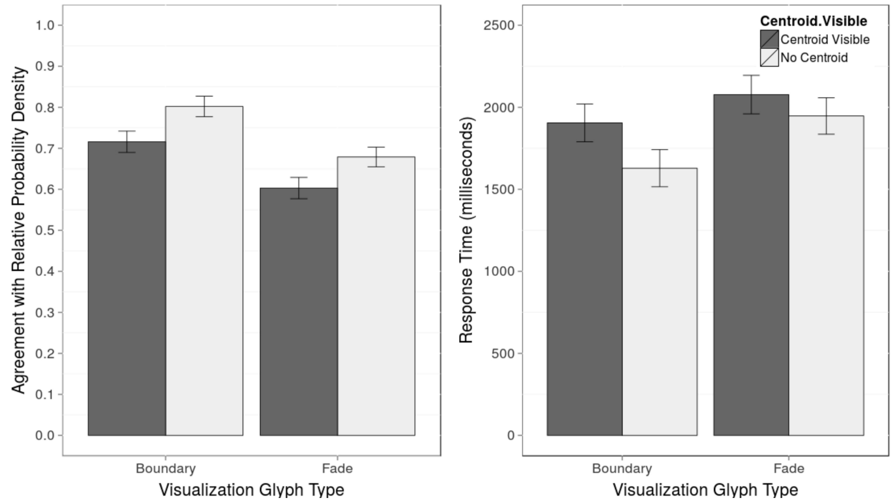


Figure 6.: (a) Response based on agreement with *relative probability density* split by glyph type and (b) Response times in milliseconds split by glyph type.

3.4.2. Effects of Visualization on Response Times

Figure 6b shows average response times across trials for participants who viewed the four different glyphs. A 2 (glyph) by 2 (centroid) ANOVA on these data indicated a significant effect of visualization, with shorter response times for the circle visualizations than for the Gaussian fade visualizations, $F\ (1, 106) = 4.60$, $p = .034$, $\eta_p^2 = .04$ (see Figure 6b). There was also a trend for longer response times when the centroid was marked in the display, $F\ (1, 106) = 3.16$, $p = .08$, $\eta_p^2 = .03$. These data address our fourth research question and indicate that there was no speed-accuracy trade off and conditions that were associated with better performance (i.e., circle glyphs and no centroid marking) were also associated with faster response times.

3.4.3. Effects of Relative Probability and Heuristics on Performance

Finally, we assess whether overall accuracy rates and response times across trials were sensitive to the actual difference in probability of the point being in the two distributions. Specifically, we tested the predictions that judgments would be faster and more accurate for trials in which the probability of the known point being in one of the distributions (visualized as a blue circle or fade) is much higher than its probability of being in the other distribution. The measure of difference in probability that we used was the *log difference in probability density (LDPD)* and was calculated by taking the absolute value of the difference in the probability density value of the point in each of the distributions ($p_{u1}(x, y)$, $p_{u2}(x, y)$) and then taking the natural logarithm of the result (Equation 2).

$$LDPD = ln(|p_{u1}(x, y) - p_{u2}(x, y)|) \tag{2}$$

As Table 2 shows, log difference in probability density was significantly correlated with both observed accuracy and response times across trials and this correlation was particularly high with response times, indicating that participants were faster to respond on trials for which the difference in probability of the known point being in the two distributions was greater.

The probability difference was higher for trials on which the distance heuristic predicted

Dependent Measure	Predictor	Zero-order Correlation	Standardized Coefficient (Beta)	t(31)	p
Overall Accuracy	Distance heuristic	0.63	0.76	7.81	<.001
	Size heuristic	0.53	0.39	3.97	<.001
	Probability difference	0.51	.07	0.64	n.s.
					Adjusted $R2 = .76$
Average Response time	Distance heuristic	-.46	-.22	2.03	.052
	Size heuristic	-.55	-0.29	2.61	.015
	Probability difference	-.81	-0.6	5.08	<.001
					Adjusted $R2 = .71$

Table 2.: Results of the regression analysis. Overall Accuracy and Average Response Time as predicted by Distance and Size heuristics as well as log difference in probability density.

the correct answer ($r = .39$) and the size heuristic predicted the correct answer ($r = .43$), so the effects of probability difference might reflect use of the heuristics. Moreover, both heuristics predicted relative accuracy across trials (see Table 2). To examine the combined effects of these two heuristics and the log difference in probability density on observed accuracy, we conducted a multiple regression analysis in which average accuracy across trials was simultaneously regressed on log difference in probability density, whether the distance heuristic predicts the correct answer for each trial and whether size heuristic predicts the correct answer for each trial. As shown in Table 2, this analysis indicated significant effects of both the distance and size heuristics on the observed accuracy across trials, whereas the effect of log difference in probability density was not significant after accounting for these two variables. Together the variables accounted for 76% of the variance in accuracy, indicating that rate of correct responses across trials was primarily predicted by the two heuristics.

We conducted a similar regression with response times across trials as the dependent variable and the same predictors. As shown in Table 2, in this analysis, probability difference and the size heuristic were significant predictors, whereas the contribution of the distance heuristic was marginal. Together the variables accounted for 71% of the variance in response times. A possible interpretation of the results of these regression analyses is that participants are sensitive to the difference in probability so that this strongly predicts their response times, but when the difference in probability is small, they resort to simple heuristics when judging which smartphone gave the better reading of the known location.

4. Discussion

In this study we employed two forms of visualization, the current Google *blue circle of positional uncertainty* as well as a Gaussian faded glyph, both with and without centroid markings, in order to investigate how different visualizations affect users' understanding of positional uncertainty. The results of this study indicate that the way in which uncertainty is visually depicted clearly influences judgments of positional uncertainty. Presented with the task of determining the more accurate smartphone display, individuals adopt heuristics. The specific heuristics adopted are dependent on how the geospatial data is visually presented to them. Specifically, both glyph type and visibility of centroid are instrumental in the adoption of different heuristics. The presence of the centroid leads to a distance-based heuristic while a uniformly opaque circle with border glyph leads participants to adopt a containment heuristic.

An interesting finding of this study is that the uniformly opaque circle with no visible centroid produced the most accurate and fastest judgments. This result was contrary to our predictions and to the *faded glyph* recommendations from previous analyses of methods for visualizing uncertainty (MacEachren *et al.* 2012). One possible explanation for this result is that the lack of visible centroid and uniform opacity of the circle glyphs made the uncertainty of the distribution more salient. The uniform circle also led to increased use of the *containment* heuristic. This is a particularly useful heuristic, as the circle shows the 95% confidence interval so that when the known point is inside one blue region but outside the other, use of this heuristic will predict the correct answer 95% of the time.

While we assumed that the more complete visual representation of positional uncertainty provided by the Gaussian fade glyph would be more effective, it in fact produced less accurate and slower judgments. We propose that both the Gaussian fade and the inclusion of the centroid made the center of the distribution more salient, causing participants to prefer the *distance* heuristic over *size* and *containment*. Glyphs that combined the visible centroid and the Gaussian fade caused the center of the distribution to be particularly salient (see Figure 2). This increased saliency of the center lead to adoption of the *distance* heuristic, which is an example of a single reason *Take The Best* heuristic (Todd and Gigerenzer 2007) which ignores the displayed uncertainty.

Given the results of the study, including the heuristics reported by the participants, we argue that completeness of the visual display of uncertainty information, as it pertains to this specific task, is less important than using visual cues that make the uncertainty information salient. In other words, simply displaying the confidence interval as a uniformly opaque circle is sufficient for conveying the concept of uncertainty while the arguably more detailed depiction of uncertainty visualized via an opaque to transparent *fade* is not necessary and potentially misleading. This reflects the findings of Smallman & St. John (2005) and Hegarty *et al.* (2012) that point to simpler, less complex visualizations being more effective in spatial decision making.

4.1. *Limitations & Future Research*

A potential limitation of the present study is that we did not explain the conventions of the displays to the participants, because our goal was to study the intuitive understanding of these displays. It is notable that despite this, overall, participant accuracy was better than chance. This suggests that participants do take uncertainty into account and that they are sensitive to differences in probabilities. An important issue for future research is to study how judgments and the adoption of heuristics might change if the conventions of the display are explained.

Another limitation is that there was an apparent size discrepancy between the uniformly opaque circle glyphs and the Gaussian fade glyphs. While statistically the glyphs represent the exact same underlying data, the nature of an opaque to transparent Gaussian fade is that it will *appear* to represent a smaller region than its circular counterpart. The concern is that this may have impacted decisions made by participants during the study. Ongoing research in our laboratory is focusing on psychophysical techniques to match the apparent size of the glyphs. Preliminary results from an experiment that replicated this reported study, but with matched glyph size, indicate that the results do replicate, implying that differences in the apparent size of the glyphs are unlikely to be a significant factor.

Furthermore, it is important to point out that the eight known locations selected for each of the distributions pairs were not randomly selected, but rather sampled to maximize discrimination among the heuristics. Future work in this area should analyze the actual cue validity of the heuristics when known locations are sampled randomly from the space. Lastly, it should be made apparent that *size* was confounded with *opacity* for uniformly opaque circular glyphs. This is redundant coding which may or may not have impacted the results of the study.

A final limitation is that our participant group was quite homogeneous, in that all of the participants were college students, almost all owned smartphones and almost all had used navigation applications. An important direction for future research will involve testing of a more heterogeneous sample of participants and examination of knowledge and familiarity effects.

4.2. *Conclusions*

The recent increase in location-based mobile mapping applications has made visual indicators of self-location (e.g., *Google's Blue Dot*) ubiquitous in our location-enabled society. This increase in smartphone usage has given rise to questions concerning the uncertainty associated with visually representing one's geospatial position. Given the wide variety of location-enabling technologies in use today, each with its own inherent uncertainty, mobile mapping providers have elected to introduce geovisual representations of this positional uncertainty.

Overall, the results of this study find that the visualization that produces the most accurate (in terms of probability) response is an opaque circle shown without a visible centroid. This implies that while a Gaussian faded glyph may present a more complete depiction of spatial uncertainty, basic statistical data portrayed through simple visualizations, such as uniform opacity and a border at the 95% confidence interval, result in more accurate uncertainty judgments.

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Appendix: Experimental Instructions

Hello and thank you for participating in the study. Today you will be answering questions comparing locations on maps similar to a GPS navigation app for a smartphone.

GPS navigation apps always give you a view of your current location, often shown by a moving blue dot as you move around the environment. However, the estimate of your current location is not usually exact, due to a number of different factors such as availability of satellite readings at your current location, and the methods used by different smart phones to estimate your location from the satellite readings. As a result, two different smart phones might give you different readings of your current location. They might also differ in the amount of uncertainty about your current location.

The images you will see in this experiment represent location readings from two smartphones. Both smartphones present the readings with a known amount of uncertainty represented by the blue circle. For each trial, you will be asked to indicate which smartphone produced the more accurate location reading, taking into account the uncertainty of the reading. The size of the blue circle represents the amount of uncertainty in the location reading: larger circles represent greater uncertainty. For each trial, suppose you and your two friends are all in the exact physical location 'X'. Both smartphones are about equally accurate on average, however only one smartphone produces the more accurate location reading for each specific location.

Suppose you and your two friends with smartphones are all currently at the known location 'X' marked on the map. The location readings from friend A and friend B's smartphones are displayed in the left and right images, respectively. Please compare your known location to each of your friend's smartphone GPS readings to decide which smartphone produced the more accurate location reading for that location.

During the trials, please work as quickly and accurately as possible. Press the space bar to begin.